PHYS 2020: Midterm 2 practice

Reading: Purcell & Morin, Chapters 2 & 3. Problems with stars (*) are more difficult.

Problem 1: Consider an infinite wire coincident with the z axis, with uniform linear charge density λ. What is the potential difference ϕ_{21} between the following pairs of points:

(a) $\vec{r}_1 = (d, 0, 0)$ and $\vec{r}_2 = (0, d, 0)$?

(b) $\vec{r}_1 = (d, 0, 0)$ and $\vec{r}_2 = (3d, 4d, 2d)$?

Problem 2: How much work does it take to move a point charge q from $\vec{r}_1$ to $\vec{r}_2$ in problem 1?

Problem 3: Consider an infinite hollow cylinder with radius R and surface charge σ. What is the potential $\phi(s)$? Assume that $\phi = 0$ at $s = 0$. Sketch your result.

Problem 4: Using your result for $\phi(s)$ from problem 3, compute $-\vec{\nabla}\phi$ and show that it reproduces the expected electric field $\vec{E}$.

Problem 5: Consider a circular wire ring of linear charge density λ and radius R. What is the potential $\phi(z)$ at a point located at a distance z directly above the center of the ring?

Problem 6: Consider a thin circular disk with surface charge density σ and radius R. What is the potential $\phi(z)$ at a point located at a distance z directly above the center of the disk?

Problem 7: Consider two infinite planes parallel to the x-y plane, separated by a distance d. Let the upper plane have uniform surface charge density σ and the lower plane have surface charge density $-\sigma$. What is the potential difference $\Delta\phi$ between the upper and lower planes?

Problem 8: Consider a parallel plate capacitor formed from two parallel conducting disks, both with radius R and separated by a distance d. Suppose the upper plate has charge Q and lower plate has charge $-Q$, both uniformly distributed. What is the capacitance C? *Hint:* You may work in the limit that $d \ll R$, treating the two plates as infinite.

Problem 9*: For the setup in **Problem 8**, determine the potential difference ϕ between the two plates *without* assuming $d \ll R$. *Hint:* use your result from **Problem 6** to determine the potential from each disk individually along the normal line bisecting the disk center. Then use the superposition principle to obtain the total potential. Show that your result agrees with the result from **Problem 7** in the limit $d \gg R$. *Answer:* $\frac{\sigma}{\epsilon_0}(d + R - \sqrt{d^2 + R^2})$.

Problem 10: Using the result from **Problem 9**, determine the the capacitance C for two parallel disks of radius R and separation d, again without assuming $d \ll R$. Show that your result agrees with your result from **Problem 8** in the limit $d \ll R$.

Problem 11: Consider a finite thin wire of length L and uniform charge density λ. What is the potential at a distance x from the midpoint of the wire along its perpendicular bisector? *Answer:* $\phi(x) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{\sqrt{1+4x^2/L^2}+1}{\sqrt{1+4x^2/L^2}-1}\right)$. Verify that the wire looks like a point charge for $x \gg L$.

Problem 12: The Rutherford model for an atom consists of a point charge nucleus of charge $+Ze$, surrounded by the electrons, which have a uniform spherical charge distribution with radius R and total charge $-Ze$. (Z is the atomic number.) What is the electric field $\vec{E}(r)$ and potential $\phi(r)$ inside the atom ($r < R$)? Sketch your results, and then check that $\vec{E} = -\vec{\nabla}\phi$.

Problem 13: Suppose an electric dipole is placed within a cavity inside a neutral conductor. Would an external observer measure the conductor to have an electric dipole moment?

Problem 14*: A total amount of positive charge Q is spread nonuniformly onto a nonconducting, flat, circular annulus of inner radius R_1 and outer radius R_2. The charge is distributed so that the surface density is $\sigma = k/s^3$, where s is the distance to the center of the annulus and k is a constant. Determine k according to the condition that the total charge is Q. Next, determine the potential ϕ at the center of the annulus, assuming $\phi = 0$ at infinity. *Answer:* $\phi = \frac{Q(R_1+R_2)}{8\pi\epsilon_0 R_1 R_2}$.

Problem 15: Which of the following electric fields $\vec{E}$ represents an electrostatic field in vacuum? Justify your answer. In the following expressions, a is a constant.

(a) $\vec{E} = a(x, -2y, z)$

(b) $\vec{E} = a(y+z, x+z, x+y)$

(c) $\vec{E} = \frac{a}{x^2+y^2+z^2}(x, y, z)$

Problem 16: Consider a nonconducting hemisphere of radius R and uniform charge density ρ resting flat side down on an infinite conducting plane. What is the surface charge density σ on the plane at the point located just under the center point of the hemisphere?

Problem 17: Purcell & Morin, 3.20.

Problem 18: Purcell & Morin, 3.55.

Problem 19: Purcell & Morin, 3.56.

Problem 20: Purcell & Morin, 3.68.